

Empowering Maths Learners (Generic Teachers' Notes)

About the exemplar activities:

These exemplar activities were designed, trialled and edited by members of the TMSJ Network: Graeme Austin, Minnie Gloor, Jane Goodland, Christina Mio, Helen Thouless, Ruth Wheatley, Pete Wright.

The Fraction Flags activity was based on an activity which was developed in the [Visible Maths Pedagogy](#) research project involving Alba Fejzo, Tiago Carvalho and Pete Wright.

The three exemplar activities are based on a teaching model that is designed to stimulate discussion and critical reflection on existing practice, and to encourage teachers to try out pedagogical approaches that might be considered more empowering. The model should be used as a prompt for pedagogical thinking and as a starting point for planning. It should not be followed prescriptively, and you may wish to adapt it to your own context and to your own students' needs.

These Generic Teachers' Notes outline a structure, which has six learning phases, that could be spread across several (perhaps 2 or 3) lessons. We anticipate that the generic structure could be applied to teaching activities across all educational phases. The Generic Teachers Notes explain the rationale for each of the six learning phases and guidance on how each phase might be organised in the classroom. You could use these notes in designing your own teaching activity for empowering learners.

There are Specific Teachers Notes provided for each of the three exemplar activities:

- **1. Border Patterns** (targeted at students aged 5-7)
- **2. Fraction Flags** (targeted at students aged 11-14)
- **3. Unearthing SUVAT Equations** (targeted at students aged 16-18)

You could use the Generic Teachers' Notes, along with the Specific Teachers' Notes, to try out one of the exemplar activities, before going on to apply the model in designing your own teaching activities.

Mathematics aims:

The activities aim to:

- Develop students' problem-solving capabilities and mathematical agency.
- Give all students a voice in generating and communicating mathematical knowledge and ideas.
- Develop a community of mathematics learners in which there is a shared responsibility for learning and all individual contributions are valued.
- Encourage students to be creative, engage with higher order thinking, and reflect on their own learning (metacognition).
- Challenge (teachers' and students') pre-conceived ideas about where authority for generating knowledge in the mathematics classroom lies.

Social Justice aims:

The activities aim to:

- Employ collaborative, discursive, problem-solving, and problem-posing pedagogies, which promote mathematical sense-making and the engagement of all learners with maths (TMSJ aim 1).
- Facilitate mathematical investigations that develop learners' individual and collective agency, enabling them to take part in future social action for the public good (TMSJ aim 3).
- Challenge common myths surrounding school mathematics, expose processes that lead to the marginalisation of learners, and open up to scrutiny what it means to be successful (TMSJ aim 4).

The six learning phases:

Phase 1: Reviewing prior knowledge

The purpose of this phase is to review the prior knowledge students might find useful in generating new knowledge. Carefully consider the mathematical ideas that might be useful to students and how these connect to each other. It might be useful to record these in a mapping diagram.

Here is one possible approach:

- Provide a series of relatively closed questions or tasks that students are likely to have come across before, and which cover the ideas you have identified above.
- Encourage students to have a go at these on their own to begin with.
- Then get students to share and discuss their responses with others (in pairs or small groups).
- You could use the following to prompt discussions:
 - *Did you get the same answers as others? If not, why not?*
 - *Did you use the same methods as others?*
 - *What are the advantages/disadvantages of different methods?*
 - *What did you find easiest and what did you find most difficult?*
 - *What important mathematical ideas do you think I want you to know?*
- Finally, ask students to summarise the key mathematical ideas that they engaged with. This could be done in (pairs/small groups) through displaying posters or oral presentations to others.

Rationale for Phase 1

The questions/tasks that you choose will prompt students to address relevant content. However, this approach emphasises the meaning that students have associated with the questions/tasks rather than merely recalling facts and procedures. It will also provide opportunities for you to assess the depth of their understanding of prior knowledge and identify any misunderstandings/misconceptions.

Phase 2: Generating ideas

The purpose of this phase is for students to generate new knowledge for themselves, by developing the mathematical ideas they have reviewed in Phase 1. It is important to establish a balance here between encouraging students to identify their own questions to explore and guiding them towards lines of inquiry that are likely to be productive. Try to avoid being too leading (however tempting this may be).

Here is one possible approach:

- Provide students with a open-ended and interesting prompt that will allow them to identify their own lines of inquiry (such as those in the exemplar activities – see Specific teachers' Notes).
- Ask students to describe what they see (even if it appears to be 'obvious'). Encourage them to express their ideas using their own words (accept informal language for now). Encourage students to build on each other's ideas.
- You could use the following to prompt discussions (some questions may be more relevant depending on the prompt):
 - *Have you seen anything like this before? What does it remind you of?*
 - *Can you see a pattern? Can you describe it in your own words? What could come next?*
 - *What is similar? What is different?*
 - *What stays the same? What changes?*
- Record each students' initials next to their response. If one student adds to the response of another student, then record both sets of initials.
- Record all responses accurately, taking care to use the words students have used themselves (avoid paraphrasing in your own words). Write in shorthand or note form if necessary (perhaps due to limited space/time).

- Display all responses simultaneously where possible (making use of multiple boards or posters).
- Encourage students to ask each other questions to clarify or develop the ideas further.
- Now ask students to consider what questions they could pose that would allow them to develop the ideas further. Acknowledge all ideas and then discuss and agree which questions might be the most productive to explore together as a class.
- Here are some questions you could use to prompt discussion:
 - *Can you create your own example?*
 - *How can you build on this idea?*
 - *What questions could you ask about it?*
 - *How might we go about answering these questions?*

Rationale for Phase 2

This is a crucial phase for establishing that the authority for generating new knowledge does not rest exclusively with the teacher. The teacher might provide an appropriate initial prompt that enables students to follow their own pathways, whilst increasing the likelihood that these pathways will be productive (see the work of Andrew Blair on [‘guided inquiry’](#) for how to use such prompts). Displaying multiple responses will emphasise that everyone’s ideas should be valued. It is important to recognise and acknowledge students’ own words that they use to describe their ideas. Including students’ initials reinforces their sense of ownership over the ideas and knowledge that they contribute, as well as facilitating subsequent discussion and questioning of each other’s ideas. The process of negotiating and agreeing on questions to explore further will reinforce students’ sense of agency, even if their own ideas are not those that are taken up. It will also promote the development of a shared responsibility for the learning of all members of the group. Identifying similarities and differences draws on the theory of ‘variation’ and enables students to notice ideas for themselves by allowing them to focus on the key mathematical concepts (see the work of [Anne Watson and John Mason](#) for more details).

Phase 3: Developing ideas

The purpose of this phase is to provide opportunities for students to work independently (from the teacher) in exploring the questions identified in Phase 2. The teacher should aim to play a facilitative role as far as possible, only intervening when a prompt/question might be useful or necessary.

Where possible, students should work collaboratively in groups so that they can share and discuss their ideas and develop their skills in communicating and presenting a mathematical argument.

Here is one possible approach:

- Carefully group students in a way that will allow less confident and slower-processing students to participate fully (e.g. mixed attainment groups).
- Encourage students to explain their ideas to each other (rather than stepping in to explain yourself) and to ask others to share their reasoning and thoughts for how to go about tackling the questions.
- Circulate amongst groups and look out for opportunities to ‘assign competence’ (e.g. by praising a contribution) to students who generally lack confidence or engagement in mathematics.
- Ask students to share a ‘good idea’ that someone else has suggested (this may help celebrate those who have contributed creative thinking but don’t always speak up themselves).
- Use the following questions/prompts to facilitate group discussions/learning:
 - *That’s an interesting idea, can you say more about it?*
 - *Does everyone agree with ...? Why? Why not?*
 - *What would happen if ...? [might involve changing one variable]*
 - *What is it that we’re trying to find out?*
 - *Talk me through how you did this.*
 - *Can you explain to others in the group why ...?*
 - *Is there another way of doing it? ... thinking about it?*

- Allocating roles to group members (e.g. ‘chair’, ‘scribe’, ‘spokesperson’, ‘timekeeper’) can be an effective way of ensuring everyone contributes and feels an integral part of the group. Be prepared to explain what each role entails, e.g. it is the chair’s job to make sure everyone contributes and everyone is heard. It is a good idea to rotate the roles between different students, so everyone gets the chance to experience each role at some point. Alternatively, you can deliberately allocate roles to students you feel would benefit, e.g. allocate the role of ‘scribe’ to someone who needs to develop their skill in listening carefully to others.
- Use mini plenaries to bring groups back together to summarise what they have found out so far, share ideas with other groups and tease out further ideas.
- Make sure that each group has a clear aim for this phase, for example, students might be working towards producing a poster, a board presentation or an oral presentation of their responses to the question(s). If appropriate, you can encourage different groups to work on different questions.

Rationale for Phase 3

Students who are capable of working collaboratively in other subjects often struggle to do so in mathematics. This reflects common myths that doing mathematics is an individual endeavour and that it is all about obtaining the correct answers to closed tasks in as short a time as possible, rather than working collaboratively, discussing ideas and developing arguments. Therefore, be prepared to guide students on how to work effectively in groups on a mathematics problem and to challenge these myths as and when opportunities arise. Note that allocating roles and assigning competence are both aspects of ‘complex instruction’ (see [Jo Boaler’s work](#) for more details). Mathematical knowledge is only powerful if the learner has the agency to apply it to solve problems they come across in real life (most problems in the workplace are solved collaboratively by teams). Generating ‘powerful knowledge’ involves developing an appreciation of ‘disciplinary meaning’ (see [Michael Young’s work](#) for more details), i.e. how new knowledge is generated within the discipline of mathematics. This implies students should be given opportunities to engage with the processes that mathematicians employ in generating new knowledge, e.g. working in teams, hypothesising, conjecturing, investigating, generalising, justifying, communicating.

Phase 4: Formalising ideas

The purpose of this phase is to enable students to appreciate the links between their own way of describing the mathematical ideas and a more formal way of doing so using more formal mathematical language and notation. It is important to establish a balance here between encouraging students to present mathematical ideas using their own language (which should be respected) and encouraging the use of more formal language as a tool for students to express themselves mathematically. The teacher might model how to use more formal language/notation alongside more informal language/notation, highlighting the rationale for doing so. Emphasise how using formal language/notation can avoid ambiguity and generate a common means for mathematicians to communicate with each other, whilst more informal language/notation may be more useful for explaining mathematical ideas to learners and making them more accessible to non-mathematicians.

Here is one possible approach:

- Use the following questions/prompts to facilitate a whole group discussion:
 - *How might a mathematician describe this idea?*
 - *What language/notation might a mathematician use?*
 - *Why might mathematicians use more formal mathematical language/notation?*
 - *When would it be better to use formal language to describe mathematical ideas?*
 - *When would it be better to use informal language to describe mathematical ideas?*
- Highlight instances where language can be ambiguous and discuss how this can be resolved with alternative wording. Here are some examples:
 - *Everyone in the class is allocated a number (either 1, 2, 3, or 4). Say ‘Hello’ to your double.*

- *These two triangles are the same.*
- *The average class size at the school is ...*
- *The probability of it raining today or tomorrow is ...*

Rationale for Phase 4

Powerful knowledge requires learners to be able to grasp abstract/formal concepts, as well as being able to apply them in solving real life problems. Therefore, whilst recognising and acknowledging the importance of students generalising from their own experiences, these ideas need to be related to formal mathematical ideas and concepts. Students also need to be able to appreciate and use precise mathematical language if they are to be successful in gaining mathematics qualifications that open up future life opportunities.

Phase 5: Reinforcing ideas

This phase provides an opportunity for students to reinforce their learning through practice and to reflect on their learning.

Here is one possible approach:

- Provide a range of tasks/questions that offer opportunities for students to practice the new knowledge they have learned. Where possible, choose tasks/questions that highlight links and connections to other areas of the curriculum. Provide a range of routine and non-routine problems (which involve students applying the ideas to solve problems in familiar and unfamiliar contexts).
- Encourage students to solve the problems independently (only seeking support from others when they are stuck).
- Provide answers/solutions for students to check themselves and focus follow-up discussions on the methods and any difficulties encountered, rather than the answers/solutions.
- Use the following questions/prompts to encourage students to reflect on their learning:
 - *How did you know which mathematical idea to use?*
 - *What difficulties did you encounter and how did you overcome these?*
 - *What advice would you give to others in tackling these types of questions?*

Rationale for Phase 5

Practising ideas, generating fluency and developing confidence is an important aspect of learning, although it is often given too much emphasis in more orthodox teaching approaches (at the expense of developing students' agency). Providing opportunities for students to reflect on their learning is an important aspect of Assessment for Learning.

Phase 6: Deepening understanding

This phase provides opportunities for students to deepen their understanding of the mathematical ideas they have engaged with in this activity by creating problems for others to solve, considering alternative paths of inquiry, and reflecting on the teaching and learning approaches that have been used.

Here is one possible approach for creating problems for others to solve:

- Students devise their own questions/problems related to the mathematical ideas in this activity for others to solve. Encouraging them to generate questions/problems with different levels of difficulty (easy/medium/hard) is a good way of ensuring they set some challenging tasks. It is important that they try out the questions/problems for themselves (to check they are appropriate) before sharing.
- Students use pieces of card (A5 or A6) for sharing their questions. On one side they write their name, level of difficulty (or alternatively use different coloured card), and the question/problem. On the other side they should write their solution with all steps in their working shown clearly.
- Students now swap cards with others (use the level of difficulty and your own knowledge of students to ensure students are given appropriate questions/problems to tackle). Students attempt each question/problem without looking at the solution. They should write their solution in their

books (or on paper or mini whiteboards) and not on the card. When they have finished, or if they are stuck, they should turn the card over and look at the solution.

- If there is any disagreement or queries about the solution, encourage students to discuss the solution with the student who devised the question (only intervene if they can't resolve the disagreement or any misunderstanding between them).

Here are some questions you could pose for considering alternative paths of inquiry:

- *What other questions could we ask?*
- *How might we go about answering these questions?*

These questions can be used to prompt a discussion about the teaching and learning approaches employed (in all phases):

- *Why did I ask you to come up with your own questions to explore?*
- *What is the advantage in working collaboratively in a group?*
- *How could you improve the way you worked together as a group?*
- *Why is it good to use formal mathematical language as well as everyday language?*
- *Why did I ask you to create your own problems for others to solve?*
- *Why do you think I decided to ...?*

Rationale for Phase 6

Devising questions/problems for others to solve reinforces students' sense of agency and shared responsibility for learning. Problem-posing (considering alternative paths of inquiry) helps students to appreciate 'disciplinary meaning', an important aspect of 'powerful knowledge' (see above), since it is only by posing questions that new mathematical knowledge is generated. There is some evidence to suggest that more student-centred pedagogies, due to their relatively unstructured and 'invisible' nature, can disadvantage some students who are less able to recognise the teacher's intentions and how to respond appropriately to achieve success. Discussing with students about the rationale for employing different teaching and learning approaches can address this issue by helping to make the teacher's 'pedagogic rationale' more explicit to learners (see Pete Wright's work on ['visible maths pedagogy'](#) for more details).